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**THE OPTIMALITY OF SEQUENTIAL PERSONNEL  
ASSIGNMENTS USING A DECISION INDEX**

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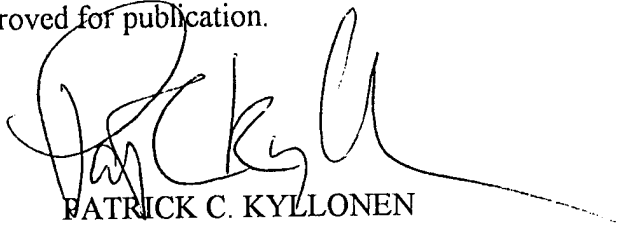
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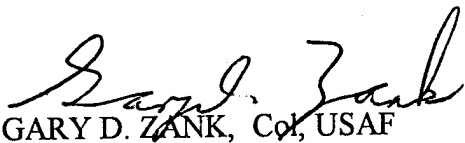
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## **PREFACE**

This work is part of the Manpower and Personnel Research Division's research program to develop a better understanding of the classification efficiency of various optimal and near-optimal personnel assignment algorithms. Work was completed under the in-house exploratory research program, Development of Technologies to Allocate People to Jobs (WU 77192024). The authors wish to thank Dr. Jacobina Skinner and Mr. Larry Looper for their technical review of this document.

# THE UTILITY OF SEQUENTIAL PERSONNEL ASSIGNMENTS USING A DECISION INDEX

## INTRODUCTION

Personnel may be assigned to jobs using either sequential or batch processes. Batch methods involve assigning a group of personnel to a group of jobs whereas sequential methods assign personnel to jobs on a "first come-first served basis." While batch processes achieve a more optimal match than sequential methods, the degree to which batch processes outperform sequential methods has not been quantified. The primary objective of this effort was to quantify the optimality of sequential assignments based on the Decision Index (Ward, 1959) throughout changing batch sizes, person-job ratios, personnel rejection rates, and personnel payoff distributions.

## BACKGROUND

The United States Air Force recruits over 30,000 enlisted personnel each year in order to fill openings in over 180 jobs. These jobs range from jet engine mechanic to security police with each having its own unique set of requirements. The Air Force must fill these jobs in order to perform its primary mission of exploitation of air power in warfare. It is desirable that each job is filled with the person best suited for that job to produce the most capable Air Force.

### The Personnel Assignment Problem

The goal when matching people to jobs is to maximize the overall "payoff" that results from the sum of all person-job matches. The "payoff" from any person-job match may be defined as the inherent "value," "utility," or "worth" associated with placing an individual in a certain job (Hendrix, Ward, Pina, & Haney, 1979). Individual payoff values are calculated by first determining various measures of interest from each prospective person-job match such as predicted technical school grade or the probability of completing a term of enlistment. Values for these measures of interest are derived from the interaction between personnel characteristics and job attributes. The measures of interest are then combined using multiattribute decision modeling techniques to yield one measure of payoff for each potential person-job match (Hendrix, Ward, Pina, & Haney, 1979).

The association of a payoff to each possible person-job match creates a payoff matrix consisting of the payoffs from the assignment of each person to each job. The following example shows a payoff matrix for 3 people and 3 jobs.

	Job 1	Job 2	Job 3
Person 1	8	7	6
Person 2	5	1	0
Person 3	6	4	1

## Linear Programming Approach

A solution to the personnel assignment problem with linear programming is possible because the problem involves the assignment of limited resources (people) to jobs in an optimal manner (Hillier & Lieberman, 1990). The example problem involving 3 people and 3 jobs shown on the previous page may be structured as the linear programming problem:

Let  $x_{ij}$  = a decision variable associated with the assignment of person  $i$  to job  $j$ . (The solution for the example problem will yield:  $x_{ij}=1$  if person  $i$  is assigned to job  $j$  and  $x_{ij}=0$  if person  $i$  is not assigned to job  $j$ .)

Using payoff matrix values as coefficients,

$$\text{Maximize Total Payoff} = Z = 8x_{11} + 7x_{12} + 6x_{13} + 5x_{21} + 1x_{22} + 0x_{23} + 6x_{31} + 4x_{32} + 1x_{33}$$

subject to:

$$\begin{array}{ll} x_{11} + x_{12} + x_{13} \leq 1 & \swarrow \\ x_{21} + x_{22} + x_{23} \leq 1 & \swarrow \\ x_{31} + x_{32} + x_{33} \leq 1 & \swarrow \\ \hline x_{11} + x_{21} + x_{31} \geq 1 & \swarrow \\ x_{12} + x_{22} + x_{32} \geq 1 & \swarrow \\ x_{13} + x_{23} + x_{33} \geq 1 & \swarrow \end{array}$$

Supply Constraints (People supplied must be less than or equal to the number of people who are available)

Demand Constraints (At least one person must be assigned to each job, that is, the number of people assigned to each job must be greater than or equal to job demands)

and

$x_{11}, x_{21}, x_{31}, x_{12}, x_{22}, x_{32}, x_{13}, x_{23}, x_{33} \geq 0$ . This is a non-negativity constraint which insures that all  $x_{ij}$  values are  $\geq 0$  to facilitate a meaningful linear programming solution.

This sort of problem may be solved using Danzig's simplex method (Wagner, 1969). The solution using this method for the example problem yields values for all  $x_{ij}$  that maximize the objective function ( $Z$ ). These  $x_{ij}$  values for all terms produce an objective function  $Z = 15$  for the example problem.

$$(x_{11}, x_{12}, x_{13}, x_{21}, x_{22}, x_{23}, x_{31}, x_{32}, x_{33}) = (0, 0, 1, 1, 0, 0, 0, 1, 0); Z = 15$$

Additionally, the simplex method may be used to find values of  $x_{ij}$  that minimize the objective function ( $Z$ ) producing a minimal solution.

More efficient solutions than Danzig's simplex method are possible by using instead a transportation simplex algorithm that represents people as supplies and jobs as demands (See Fig 1). Additionally, the transportation representation is also preferred because it allows for the representation of jobs with openings for more than more individual. Such a representation is preferred since classification of new Air Force recruits involves assigning them to one of many

identical "quotas" or school seats available for the same type of training course. (Hillier & Lieberman, 1990).

Although linear programming provides "optimal" person-job assignments, it relies on the availability of a batch of people and jobs where people must wait to be assigned and jobs must wait to be filled. This batch approach prevents recruiters from "booking" jobs on a periodic basis and prohibits jobs from being filled as they become available. These factors limit the usefulness of linear programming for assigning enlisted personnel to jobs. For these reasons, the implementation of a sequential system that assigns people to jobs as the people are available would be beneficial to the Air Force.

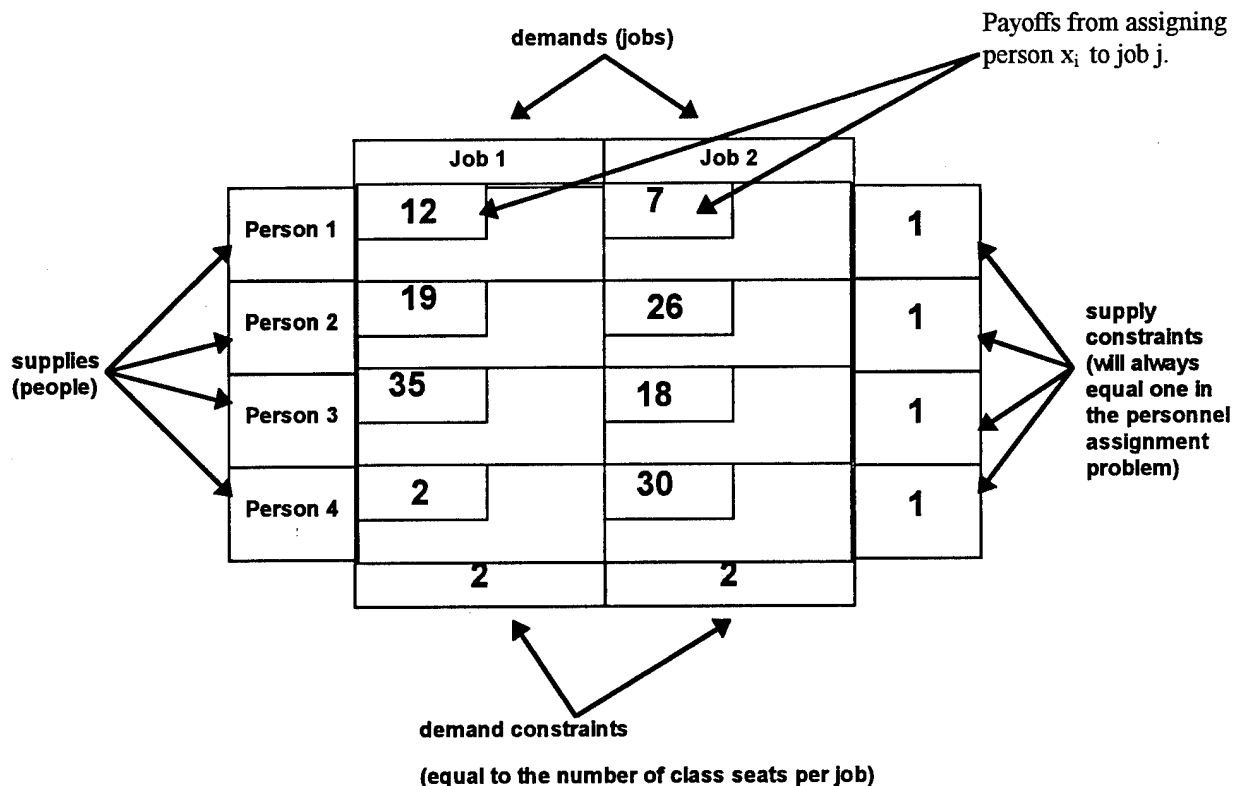


Figure 1. Example formulation of transportation problem to solve the personnel assignment problem

### Development of the Decision Index

One of the first sequential approaches to the personnel assignment problem was developed by Brogden (1946). He suggested that there was a better way to assign people to jobs than merely assigning an individual to the job for which the applicant had the highest test score. Brogden suggested a method for assigning personnel based on factors besides an individual's test scores for the determination of proper job placement. His method calculates "critical rejection scores" using distributional information from historical test scores as well as job demands. Critical rejection scores are values that must be exceeded by an individual's test scores before he/she may



be assigned to a specific job. Assignments made using critical rejection scores outperformed simple sequential assignments (Brogden, 1946).

Dwyer extended the concept of using historical "payoff" information with the method of optimal regions. (Dwyer, 1954) Further refinement of this concept introducing the "Decision Index" was made by Ward (1959).

The decision index (DI) is based on the concept that in order to assign a person to a job, job "payoff" information from other potential applicants must be considered. Batch processes such as linear programming do take into account other personnel "payoffs" when making an assignment because this information is available. In a sequential process, the specific scores of other personnel are not known, but information about the way people historically score may be estimated. This information can be used to make an assignment which optimizes the benefit from all assignments in contrast to a simple sequential strategy that maximizes benefits for a single person at a time. Since some jobs possess lower mean payoff values than other jobs, the decision index transforms individual payoff matrix cell values to account for these differences. The decision index increases the payoff of individuals with high payoffs in jobs with low mean payoff values while reducing the payoffs of individuals with low payoffs in jobs with high mean payoff values.

Mathematically, there are various implementations of the decision index. The basic form for a cell (i,j) in a payoff matrix is:

$$DI_{ij} = (\text{Cell payoff}) - (\text{job (column) effects}) - (\text{person (row) effects}) + (\text{Correction})$$

or

$$DI_{ij} = \frac{1}{n(m-1)} (mc_{ij} - c_i - c_j + c) \quad (1)$$

where:

m = number of personnel assigned

n = number of jobs available

$c_{ij}$  = assignment cell payoff

and

$$c_i = \sum_{j=1}^n c_{ij} \quad (\text{Column sum of payoffs})$$

$$c_j = \sum_{i=1}^m c_{ij} \quad (\text{Row sum of payoffs})$$

$$c = \sum_{i=1}^m \sum_{j=1}^n c_{ij} \quad (\text{Grand sum of payoffs})$$

This representation of the DI represents the mean of all (n-1)! possible assignments that could be made involving any particular cell (Ward, 1959). In a sequential assignment, where no information about the payoffs of other individuals is available, historical means are substituted.

Similarly, in a practical implementation of the decision index within a sequential system the terms  $\frac{1}{n(m-1)}$ ,  $c_i$ , and  $c$  may be removed because they will always be equal for all row values. These simplifications reduce the operational form of the DI to:

$$DI_{ij} = (c_{ij} - c_j) \quad (2)$$

### Example Application of The Decision Index

The following example shows an application of the decision index to sequentially assign a batch of 3 people to 3 jobs. The DI is not recalculated following each assignment.

The payoff matrix with computed row, column, and matrix sums is:

Job Person	1	2	3	$c_i$
1	8	7	6	21
2	5	1	0	6
3	6	4	1	11
$c_j$	19	12	7	38

Sequential assignments from this matrix without the use of a decision index yield the following assignments: person 1 to job 1; person 2 to job 2; and person 3 to job 3. These sequential assignments result in an overall "payoff" of  $8 + 1 + 1 = 10$ .

The computation of a DI matrix or "allocation matrix" using the DI equation (1) yields:

Job Person	1	2	3	$\sum_{i=1}^3 DI_i$
1	3.667	4.333	4.666	12.66
2	4.667	3.833	4.166	12.66
3	4.333	4.5	3.833	12.66
$\sum_{i=1}^3 DI_j$	12.66	12.66	12.66	38

Sequential assignments from this allocation matrix (without DI recalculation following each assignment) sends person 1 to job 3; person 2 to job 1; and person 3 to job 2 resulting in an overall "payoff" of 15. Even though sequential assignments which are optimal are possible, as was shown in this example, this result is not typical. Sequential personnel assignments that are made using a decision index generally yield suboptimal assignments due the ordered nature of sequential assignments.

An example of the application of the decision index to an assignment situation without knowledge of the historical “payoffs” for other personnel involves one person who may be assigned to three jobs.

	Job 1	Job 2	Job 3
Person 1	8	7	6

A sequential assignment system without the decision index would assign person 1 to job 1. However, a sequential assignment system that implements the decision index subtracts historical column average “payoffs” from each job “payoff.”

	Job 1	Job 2	Job 3
Person 1	$8 - 5 = 3$	$7 - 3 = 4$	$6 - 10 = -4$

If historical job “payoff” values were 5, 3, and 10 respectively, person 1 would be assigned to job 2.

### Decision Index Implementation

In order to apply the DI to sequential personnel assignments, it is necessary to first calculate average job payoff values (column means) from historical data. These column means are then subtracted from each individual’s payoff values for all jobs for which the individual qualifies. This method may be modified to find a sequential solution to a problem that can be solved in a batch. If a batch of people and jobs are awaiting sequential assignment, column means can be calculated from this set to create an allocation matrix. Jobs are allocated based upon this matrix. Recalculation of column means following each assignment may be performed to further increase the total payoff (Ward, 1959).

Other variations of this approach can be used to find sub-optimal approximations to the transportation problem. The decision index may be calculated for all values of a transportation matrix. The highest of these DI values may then be assigned. Recalculation of the decision index following each assignment improves the optimality of an assignment when this method is used. The application of the decision index to a transportation problem is a slight variation of Russell’s method that subtracts the greatest column and row payoffs from a shipping cost rather than the column and row means (Hillier & Lieberman, 1990).

Previous work by McWilliams (1970) compared several methods for approximating the solution of a transportation problem. The methods compared included the cost minima, northwest corner rule, column by column minimum, Vogel’s, Ward’s, and the  $C^q$  methods. Results showed that, of these methods, Ward’s and the  $C^q$  provided the closest approximation of an optimal result. Furthermore, Ward’s decision index method produced solutions that captured approximately 96% of the payoff attained through optimal methods. These approximations did not vary despite changing matrix sizes, or ratio of supply variables to demand variables (McWilliams, 1970). McWilliams’ results were based upon recalculation of the decision index following each assignment—an approach which produces more optimal results than assignment

without recalculation. Also, McWilliams' application of the decision index was implemented by "assigning individuals" in accordance with the relative magnitude of their highest DI payoff value. This approach differs from a sequential assignment strategy that requires assignment of all individuals in a sequential manner. McWilliams was concerned with finding near-optimal approximations to transportation problems rather than assigning individuals in a sequential manner.

## STATEMENT OF THE PROBLEM

The problem addressed by the current study concerned the degree to which sequential personnel assignments, without DI recalculation following each assignment, would approximate optimal assignments under varying conditions including: changing batch sizes, person-to-job ratios, personnel rejection rates, and personnel payoff distributions. This effort did not allow for DI recalculation following each assignment, as was done by McWilliams, because this approach would be impossible to implement in a truly sequential system. In a sequential assignment system, entire payoff matrices never exist making it impossible to recalculate DI values following each assignment.

## METHOD

Payoff matrices were generated for several experimental conditions (e.g., size, payoff distributions, etc.) using Monte Carlo techniques. Assignments were then made from these payoff matrices using five methods: optimal assignments, sequential assignments using a decision index, simple sequential assignments, random assignments, and minimal assignments. The fitness of the competing assignment methods was judged by determining how well their payoffs approached optimal values. This process was repeated multiple times for each set of conditions in order to achieve generalizable results.

### Generation of Payoff Matrices

Payoff matrices were generated to represent the resulting payoff from assignment of people to jobs. Each cell ( $c_{ij}$ ) of the matrix represents the payoff to the Air Force resulting from assignment of person (i) to job (j). For the purposes of this study, the payoffs were generated to be identically and independently distributed (IID). Since payoffs represent the intrinsic utility or worth to the Air Force arising from assigning an individual to a specific job, it is a reasonable assumption that they are normally distributed. In selected cases the distribution was truncated at the 40th percentile to approximate the Air Force Qualifying Test (AFQT) entry standard.

Various sizes of matrices were generated ranging from 5 x 5 to 50 x 50 matrices. Additionally, various person-to-job ratios were examined. Jobs with multiple openings or "class seats" were represented by the creation of identical payoffs in multiple columns. The number of identical columns generated was equal to the number of class seats available in a job. The following example shows the difference between expressing 3 people and 3 jobs (with one class seat per job) and expressing 3 people and 1 job (with three class seats per job).

	Job 1	Job 2	Job 3
Person 1	3	9	11
Person 2	5	8	1
Person 3	3	6	8

one class seat per job

	Job 1	Job 1	Job 1
Person 1	3	3	3
Person 2	5	5	5
Person 3	3	3	3

three class seats per job

In cases where there were more people assigned than jobs available, dummy jobs were created to allow  $\sum_{\text{people}}$  to =  $\sum_{\text{job seats}}$ . This condition is necessary for solution of the transportation problem and the assignment problem. (Hillier & Lieberman, 1990). The method of expressing job demands with multiple columns and then applying the assignment problem is mathematically equivalent to expressing multiple job demands in the transportation problem.

### Linear Programming Solution of Assignment Problem

In the present study, since jobs possessing multiple class seats were represented with multiple columns rather than by job demands exceeding one, this representation reduced the transportation problem to the special case of the assignment problem. Assignment problems are transportation problems that have all supply and demand variables equal to one (Hillier & Lieberman, 1990). This problem reformulation required the use of the Hungarian method for finding optimal solutions (Storm Version 2.0, 1989).

### Comparison of Methods Using Range of Values Metric (p)

In order to evaluate the relative performance of the five personnel assignment methods, a metric developed by McWilliams (1970) was used. This metric (p), which reflects the percentage of optimal solution attained by a given method, is expressed as a function of the achieved average payoff per person (a) from a given assignment algorithm; the optimal average payoff per person (o); and the minimal average payoff per person (m) using the equation:

$$p = \left( \frac{a - m}{o - m} \right) * 100 \quad (2)$$

The metric (p) will equal 100 if an optimal solution is reached and will equal 0 if a minimal solution is achieved. Other values of p express solutions that range between optimal and minimal.

Since this metric varies with the specific random payoffs that populate a matrix, multiple matrices were created for each experimental condition to create a sample upon which an expected value of p could be inferred. Also, because each matrix was composed of random variables drawn from normal distributions, the overall average payoffs per person resulting from solutions to the matrix were also normally distributed. The central limit theorem suggests that the expected value of p may be found by taking the average of a sufficiently large sample of values of p (Hillier & Lieberman, 1990). In order to ensure that the mean of a sample of 50 matrices was representative

of the true population mean, the confidence interval for the optimal values of the 5 x 5 matrix was determined. The 5 x 5 matrix was used because it contains the greatest variation in its optimal solutions among the matrices that were used causing its confidence interval to be the largest of all matrix sizes that were examined in the study. The confidence interval for the sample mean of  $p$  from a sample of 50 5 x 5 matrices at  $\alpha=.01$  was within an acceptable  $\pm .1 E(p)$ .

### Benchmarking Method With Prior Work

In order to ensure the validity of the payoff generation methods and assignment algorithms we compared these procedures with prior work by Brogden (1959). He examined the average optimal payoff that could be achieved for varying numbers of jobs and personnel rejection rates. He used IID payoffs from a  $N(0,1)$  distribution.

### Experimental Conditions

The ability of sequential assignments to approximate optimal assignments under four experimental conditions was examined. The first condition examined was changing *batch size*. Batch size refers to the size of the matrix under consideration. In a batch assignment system, this value refers to the number of personnel that are assigned in a group.

Additionally, the impact of changing *person-to-job ratios* on the optimality of sequential assignments was investigated. Specifically, the conditions examined were: fixed numbers of people and varying numbers of jobs; and fixed numbers of jobs and varying numbers of people. Values of  $E(p)$  were calculated for each condition.

Another aspect of varying person-to-job ratios dealt with examination of the effect of different *personnel rejection rates* on the optimality of sequential assignments. Matrices were generated with payoff drawn from  $N(0,1)$  distributions. These matrices were populated so that there were more people than jobs for each assignment. This was done by allocating a percentage of the columns of the payoff matrix as dummy variables. People who were assigned to these columns were considered to have been "rejected."

	Job 1	Dummy job
Person 1	65	0
Person 2	78	0

This procedure (dummy variables) was acceptable for examining batch assignments but not for sequential assignments. In sequential assignments with more jobs than people, people will be assigned to jobs until all of the jobs are filled. This method will always "reject" the last people in a batch. This problem motivated the development of "personnel qualification scores" for the implementation of sequential assignments with personnel rejection. "Personnel qualification scores" had to be exceeded by the sum of an individual's "payoffs" before that individual could be assigned. If this "personnel qualification score" was not exceeded, an individual was considered "rejected."

Personnel qualification scores were calculated by summing random variables (C) across the rows of a payoff matrix to create a new random variable (S).

$$S = \sum_{i=1}^N C_i \quad (3)$$

Cutoff scores were then calculated to reject a fixed percentage of personnel based on standard normal probabilities for the random variable S.

Finally, the effect of changing the underlying properties of the distribution from which payoffs were drawn was examined. The specific variables examined included the standard deviation and the truncation point of the *payoff distribution*.

## RESULTS

### Benchmark of Methods

The methods used in this study produced results that were comparable to Brogden's results (1959). Using payoffs drawn from a N (0,1) distribution, increasing numbers of jobs were exposed to fixed groups of people. Results showed almost identical optimal payoff values to those predicted by Brogden (See Figure 2). Similar results were seen when the methods used were compared to Brogden's results for varying personnel rejection rates (See Figure 3).

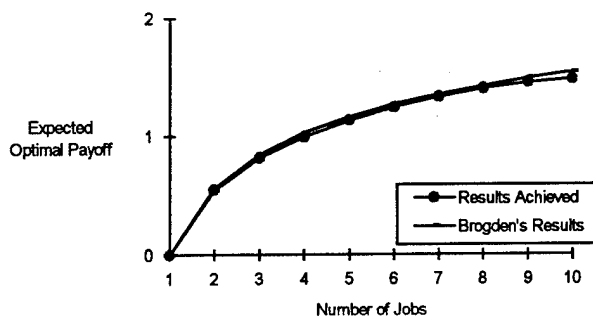


Figure 2. Benchmark with Brogden's results for changing numbers of jobs

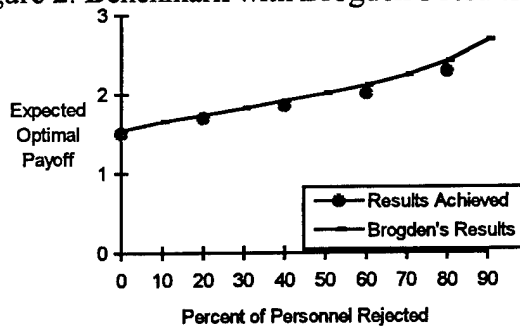


Figure 3. Benchmark with Brogden's results for changing rejection rates

## Batch Size Results

Batch sizes ranging from 5x5 to 50x50 matrices were examined. Additionally, job demands included: 1 class seat/job, 5 class seats/job, and 10 class seats/job. Each expected value was derived from a samples of 50 separate matrices. Each matrix was populated from a truncated normal distribution  $N(50,100)$ . Truncation occurred at the 40% percentile or at .27 standard deviations below the mean to represent Air Force entry standards. As expected, larger payoffs resulted from larger batch sizes (Figure 4). Further, expected p values resulting from sequential methods with and without the decision index remained relatively constant throughout changing batch sizes (about 92%). Additionally, sequential methods using the decision index were only slightly higher (<1%) than strictly sequential methods. This phenomenon was exhibited for 1 class seat, 5 class seat, and 10 class seat jobs. Increasing the batch size had no effect on p for all sequential assignments regardless of whether or not a decision index was used.

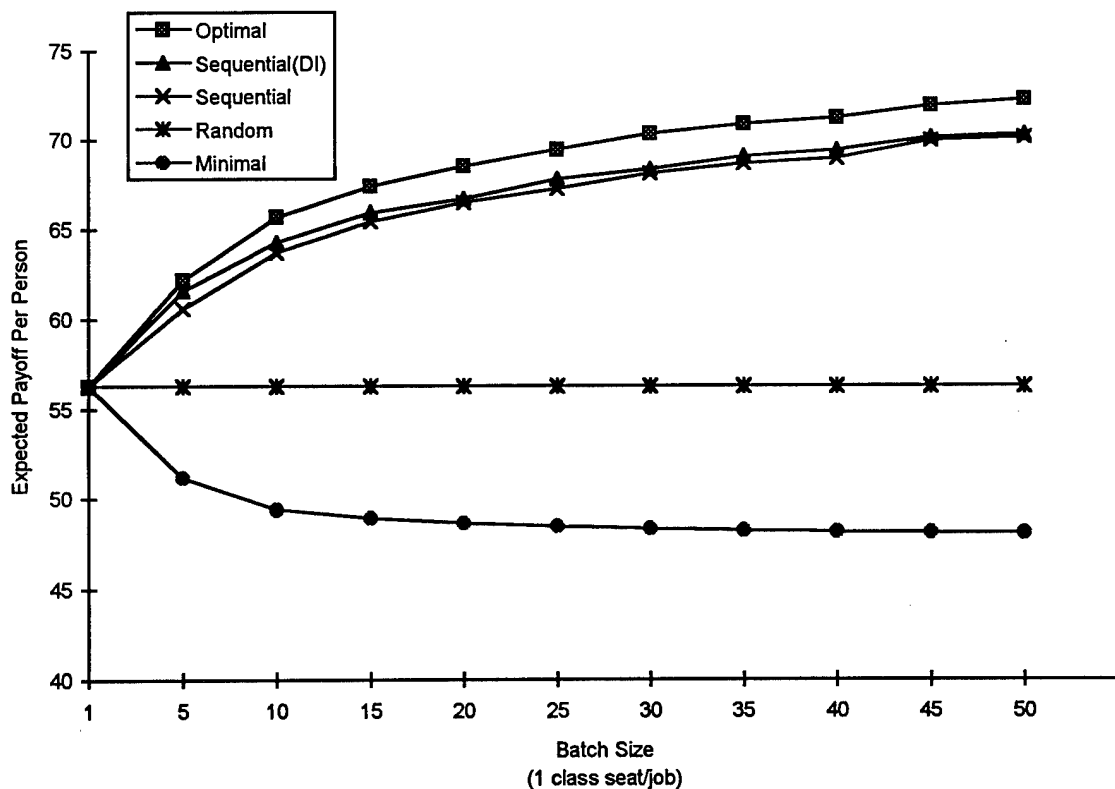


Figure 4. Effect of varying batch size on average payoff per person

## Person-Job-Ratio Results

The effect of changing the person-to-job ratio was examined by first holding the number of jobs constant and changing the number of people available. Then, the number of people available was held constant and the number of available jobs was varied. Payoffs were drawn from a truncated normal distribution  $N(50,100)$  that was truncated at the 40th percentile. The number of available people ranged from 1 to 50. From Figure 5, it can be seen that changing the number of people had no effect on the overall optimal, minimal, sequential(DI), sequential, or



random expected payoffs. As was the case with changing batch sizes, the expected value of  $p$  remained at about 92% throughout changing conditions. Similarly, sequential assignments that were made with the DI were only slightly better (<1%) than assignments made without the DI.

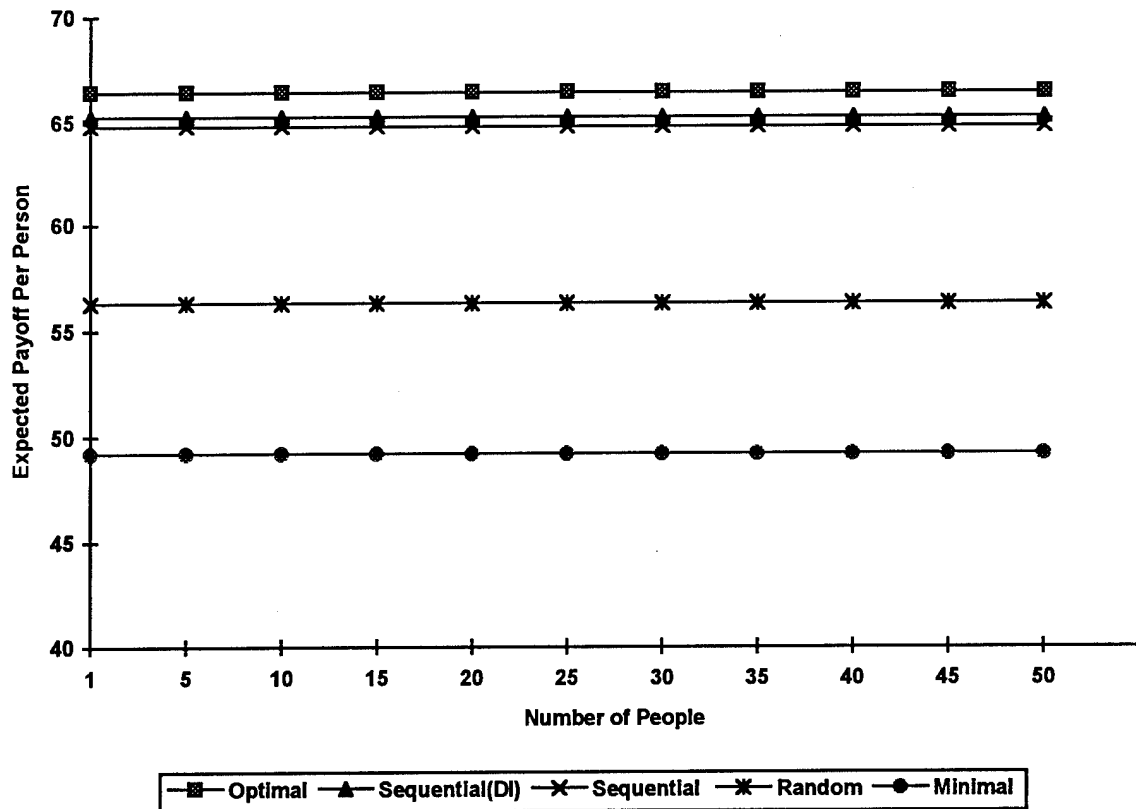


Figure 5. Effect of varying the number of available personnel on average payoff per person

Investigation into the effect of varying the number of jobs and keeping the number personnel constant was accomplished using similar procedures. The number of personnel available for assignment was held constant at 50 and the number of jobs ranged from 1 to 50. Payoffs were drawn from the normal distribution  $N(50,100)$  truncated at the 40th percentile. Results presented in Figure 6 showed an increase in the expected payoff as the number of jobs available increased. Additionally,  $p$  values for sequential (DI) and sequential assignments remained constant throughout changing conditions.

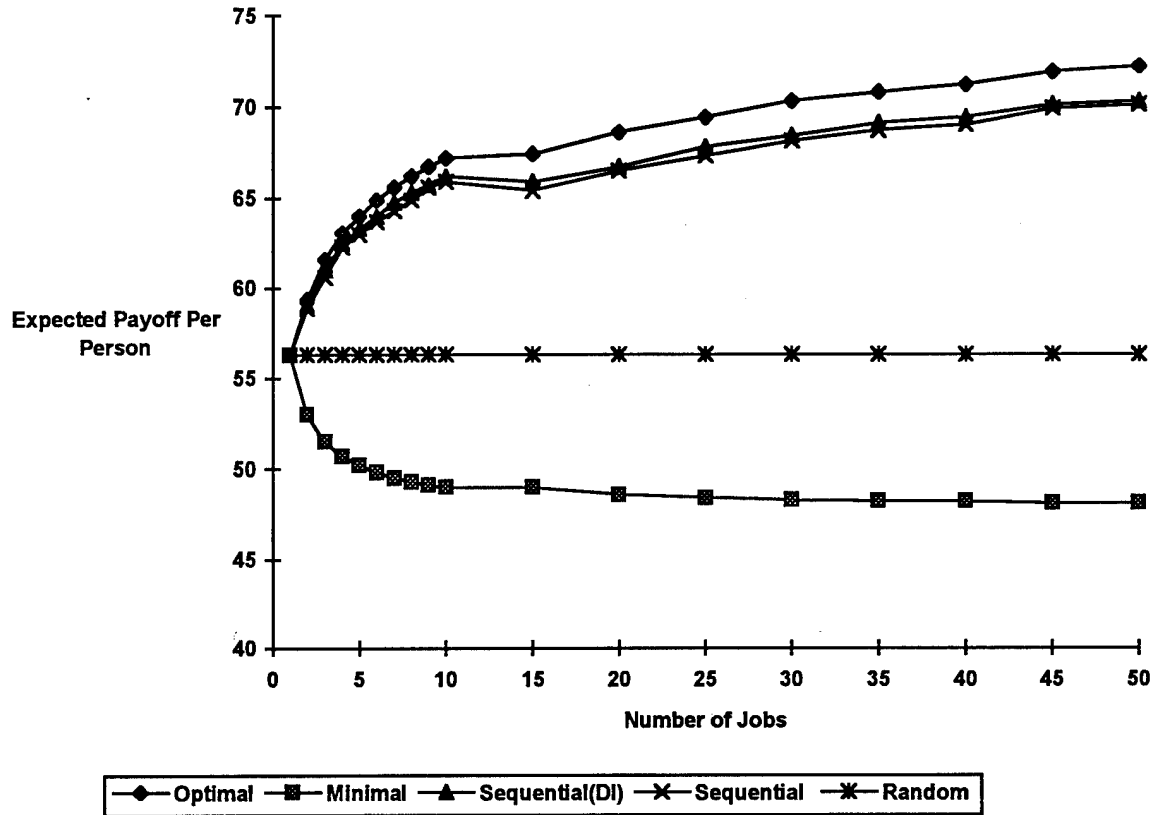


Figure 6. Effect of varying the number of available jobs on average payoff per person

### Personnel Rejection Results

Investigation into the effect of varying personnel rejection rates was accomplished by generating 10 by 10 matrices with varying numbers of columns designated as "dummy jobs." For example, a 10% rejection rate designated one column as a "dummy job." Next, the distributions from which personnel payoff values were generated,  $N(0,1)$ , were summed to formulate a random variable representing the sum of the personnel payoff scores. Rejection or qualification scores were calculated based upon normal standard probabilities. For example, the qualification score ( $r$ ) in a 10% rejection situation with 10 people and nine jobs would equal a value of  $r$  such that:

$$P\left(\frac{\sum_1^9 C}{9} < r\right) = .1$$

This value of  $r$  would be computed by first summing the payoff distributions across available jobs to calculate  $S(0,9)$ . This value of  $S$  would then be standardized and normal probability tables would be used to determine a value of  $r$  that rejects 10% of all applicants.

Results from the rejection of varying percentages of personnel as shown in Figure 7 were similar to previous results. Payoffs for sequential assignments with the decision index were

slightly higher than those for sequential assignments without the decision index (<1%). The expected values of  $p$  remained constant at 92% through rejection rates that ranged from 0 to 80%.

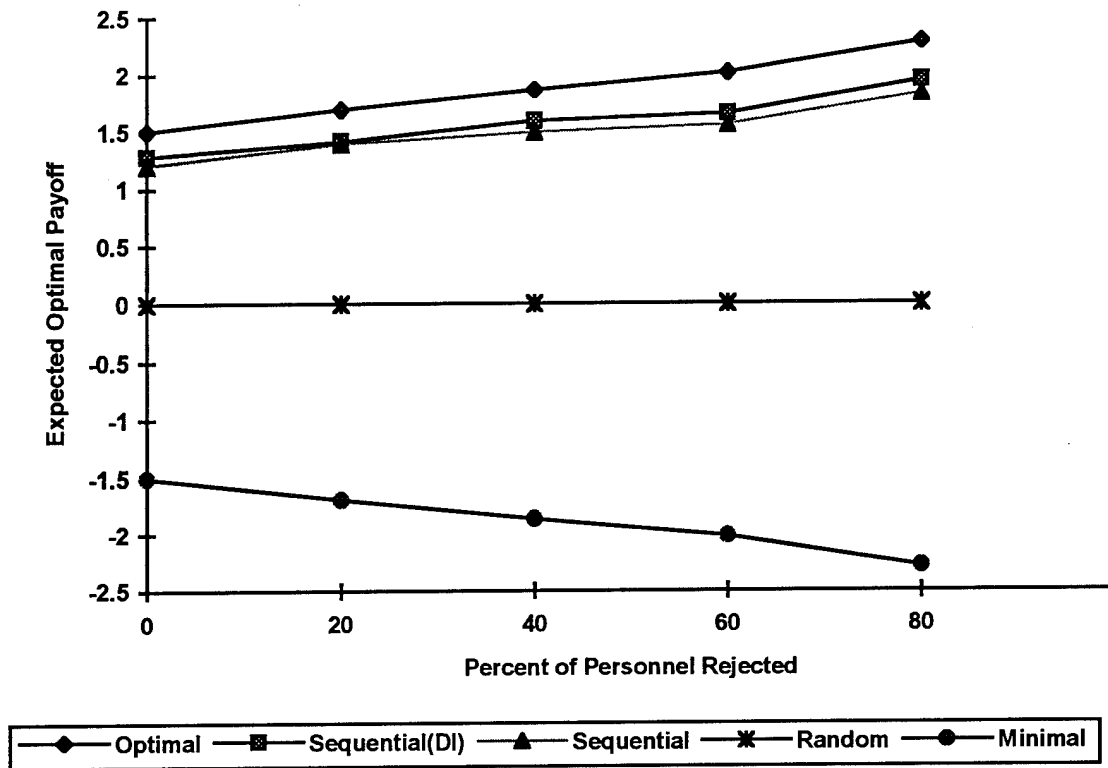


Figure 7. Effect of varying batch size on average payoff per person

### Payoff Distribution Results

The final area of investigation involved the effect of varying personnel payoff distributions on the utility of sequential assignments. The first parameter changed was the variation of the payoffs. For a fixed sample of 10 people and 10 jobs the standard deviation of the distribution from which the matrices were generated varied from 0 to 30  $\{N(50,0) \Rightarrow N(50,900)\}$ . Expected payoff values (Figure 7.) for optimal and sequential assignments showed a linear increase as payoff distribution was enlarged. Sequential assignments continued to achieve  $p$  values of approximately 92.

The effect of changing the truncation of personnel payoff distributions was examined by using 10 people and 10 jobs. Payoffs were drawn from a normal distribution  $N(50,100)$  with truncations that ranged from no truncation to truncation at the 90th percentile. Results showed increased average payoffs per person with higher truncations as indicated in Figure 8. Sequential assignment  $p$  values remained constant throughout the truncation range.

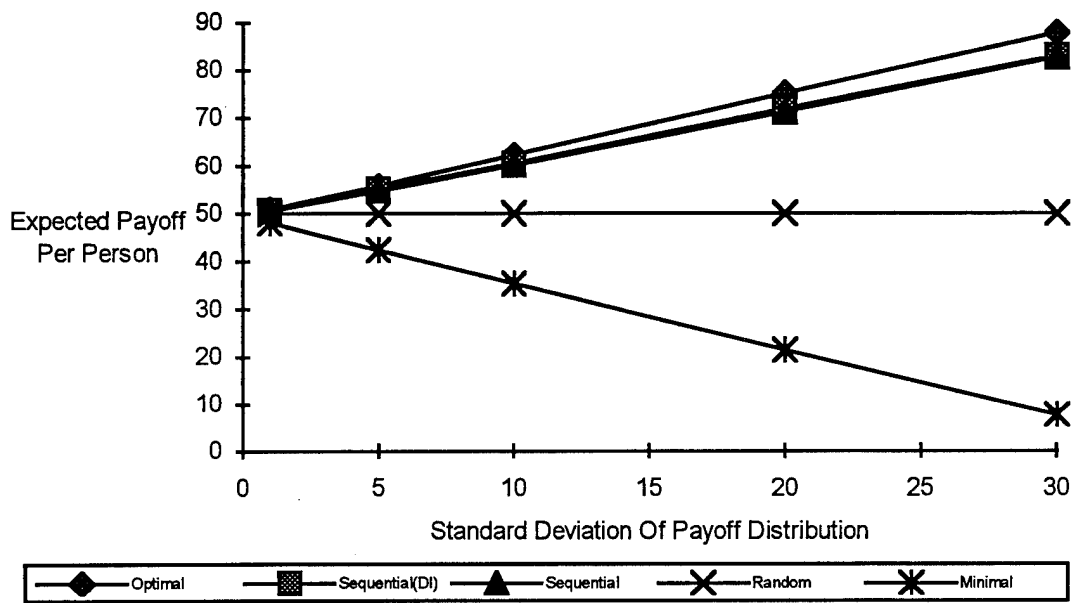


Figure 8. Effect of varying payoff variation on average payoff per person

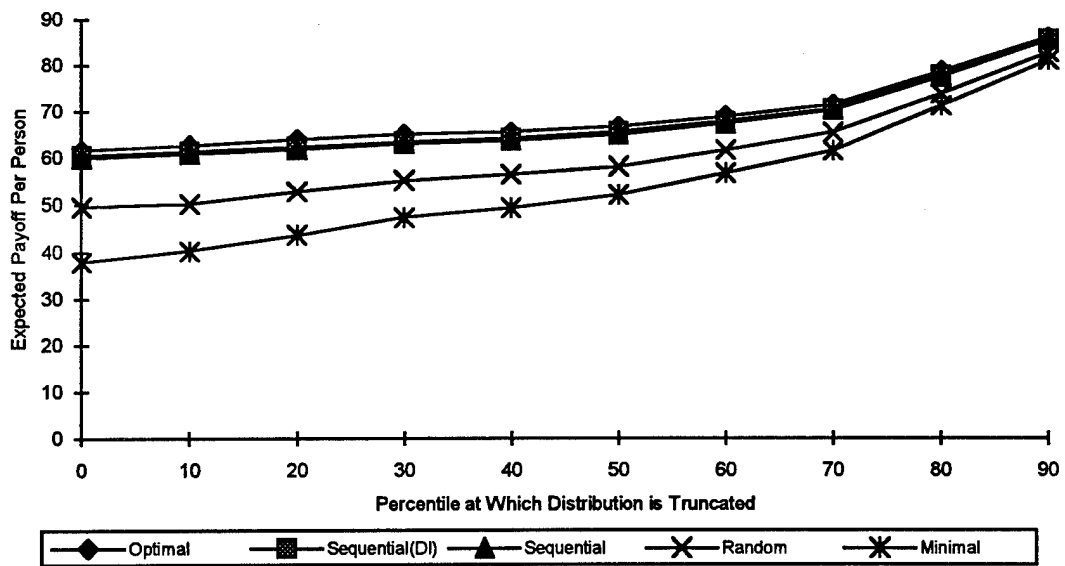


Figure 9. Effect of varying the truncation of the payoff distribution on average payoff per person

## CONCLUSION

Throughout changing conditions (batch size, person-to-job ratios, personnel rejection rates, and personnel payoff distributions), sequential assignments achieved a constant percentage of the average payoff per person compared to those attained through optimal assignments. On a linear scale that labels the minimal expected assignment value as 0 and the optimal expected assignment value as 100, sequential assignments achieved an expected value of 92%. This value is slightly higher (< 1%) for those assignments that were achieved using a decision index (without DI recalculation following each assignment). The slight differences between these sequential assignment methods can be attributed to the lack of different column means in the payoff matrix. The IID payoffs created matrices with column means that were nearly identical rendering the DI almost indistinguishable from the "highest payoff" decision rule. The slight column differences that did exist explain the slight amount that DI-assisted assignments exceeded those made without the DI. Future work should evaluate the performance of decision index sequential assignments with varying column means in the payoff matrix (Grobman, In Progress).

Although the expected values of 92% were lower than McWilliams' (1970) value of 96%, these differences are explainable. First, McWilliams applied the decision index by assigning the "person-job match" with the highest DI value first. Our methods assigned the first individual before all others regardless of his relative DI value. Additionally, McWilliams allowed for DI recalculation following each "assignment" whereas we favored the more conservative application with no recalculation. Both of these differences in methods may explain the differences between McWilliams and the present results.

Results from the study also show that even though the DI achieved a relatively constant 92% of the total possible benefits, the absolute levels of the total benefits varied dramatically with some of the study conditions (i.e., batch size, personnel rejection rates, and initial level of truncation). This could have important implications for Air Force classification, and assignment policies. For example, increasing batch size raises the overall benefits possible from assignment as would raising entry standards, and increasing the number of jobs available for fill.

Future work should investigate the difference in performance between sequential assignments made with the DI and those without the DI under conditions involving different row and column means in the payoff matrix. We think that these conditions should cause the DI to outperform strictly sequential assignments by a significant amount. Another area for future work is in the area of stochastic programming (Wagner, 1969). Stochastic programming involves adding random variables to linear programming problems to achieve expected outcomes. These methods may produce analytic solutions, in lieu of the Monte Carlo approach, to the types of theoretical research questions examined in this study. These methods may surpass Monte Carlo methods with respect to their generalizability and computational efficiency.

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